1. **Definition of the Internal Carotid Artery (ICA)**

The centerline of the ICA was extracted from medical imaging data using V-Modeler (an in-house software developed in Oshima Lab). After the 3d thinning algorithm, a fifth-order spline interpolation was applied to generate a smooth and continuous representation of the vessel centerline.

The ICA centerline can be expressed as an ordered sequence of points:

where each point:

represents the three-dimensional coordinates of the centerline.

In this study, the cumulative arc lengthis measured along the centerline starting from the bifurcation between the ICA and the middle cerebral artery (MCA) and ending at the bifurcation between the ICA and the common carotid artery (CCA). Along the centerline, the curvature and cumulative arc length are defined respectively as:

where denotes the curvature at point , which is equivalently expressed as along the arc-length parameterization. represents the cumulative arc length from the starting point of the centerline to the position of .

1. **Definition of the ICA Siphon**

From an anatomical perspective, the ICA siphon is located between the C4 and C6 segments of the internal carotid artery, as illustrated in *Figure 1*. This region is characterized by two distinct curvatures: the superior bend (B1) and the inferior bend (B2).

形状

AI 生成的内容可能不正确。图示

AI 生成的内容可能不正确。

*Figure 1. Definition of the ICA and Siphon*

The maximum curvature of the B1 bend is generally located approximately downstream from the starting portion of the ICA segment (the ICA/MCA bifurcation). In most cases, the maximum curvature at B1 corresponds to the global maximum curvature of the entire ICA centerline. The next local curvature peak downstream of B1 is defined as the B2 curvature peak.  
In this study, the arc-length positions of the B1 and B2 curvature peaks are denoted as and , respectively, while their corresponding curvature values are denoted as and , as shown in *Figure 2*.

图表, 折线图

AI 生成的内容可能不正确。

*Figure 2. The identification of the two major curvature peaks, B1 and B2, within the ICA siphon region.*

After determining the positions of the maximum curvature points corresponding to the B1 and B2 bends, the curvature distribution along the centerline was further analyzed to identify the adjacent local minima associated with each peak. These local minima were used to define the boundaries of the siphon region, as illustrated in *Figure 3*.

图表, 折线图

AI 生成的内容可能不正确。

*Figure 3. Identification of the curvature valleys defining the proximal* ()*, distal* ()*, and intermediate* () *boundaries of the ICA siphon.*

The red line in *Figure 3* represents the curvature valley located proximal to the B1 peak (proximal start), denoted as , while the yellow line indicates the curvature valley near the B2 peak (distal end), denoted as . The black line corresponds to the curvature valley between the two peaks, denoted as , which serves as the boundary separating the two bends.

The corresponding set of centerline points contained within this region is represented as:

Within the siphon region, the centerline points corresponding to the two major bends are defined as:

1. **Extraction of Geometric Parameters of the ICA Siphon**

Curvature and torsion are fundamental quantities for describing the geometric properties of a spatial curve. However, in this study, they are not used as indicators of the siphon’s geometric features. This is because curvature often exhibits nonlinear variation, making direct comparison between different cases difficult. Furthermore, the calculation of torsion is highly sensitive to local noise and numerical fluctuations, which can reduce its stability. The ICA siphon shape typically consists of two prominent bends (B1 and B2). Therefore, this study employs a geometry-based fitting method to describe its overall shape.

The detail is illustrated in *Figure 4*. In step (c), for each bending region, a segment of the centerline that represents the characteristic circular arc of the bend is first selected. In step (d), a best-fit plane is obtained by applying the least-squares method to the selected point set, which approximately fits the spatial distribution of these points. In step (e), the centerline points of the segment are orthogonally projected onto the fitted plane, and a circular arc is further fitted on the plane to obtain the representative geometric feature of the bend. Finally, in step (f), the radii of the two fitted circles, and , as well as the inter-plane angle between the two fitted planes, are determined. These parameters respectively represent the degree of curvature of each bend and the degree of non-coplanarity between the two bends in three-dimensional space.

图片包含 图表

AI 生成的内容可能不正确。

*Figure 4. Stepwise procedure for geometric fitting of the ICA siphon: (a–f) illustrate the selection of bending segments, plane and circle fitting, and extraction of curvature radii and inter-plane angle.*

* 1. **Definition of the Fitted Arc**

To perform circular fitting for each bending segment, it is first necessary to determine the portion of the centerline that represents the characteristic geometry of the vessel. In this study, the range of the fitted arc was defined based on a curvature threshold .

Taking the B1 bend as an example, the start and end positions of the fitted arc were determined by searching along both upstream and downstream directions from the point of maximum curvature . The continuous region where the curvature values satisfied was defined as the fitted arc segment.

The cumulative arc lengths corresponding to the first and last points of this region were denoted as and , respectively. The fitted arc segment was then defined as:

* ：threshold parameter (), which controls the length of the fitting range.
  1. **Definition of the Fitting Plane**

To construct the fitted plane for each bending segment, the set of points forming the fitted arc is used. This set can be expressed as:

where each point:

An arbitrary plane in space can be represented in the general form:

where the vector denotes the plane’s normal vector.

The centroid of the selected point set is given by:

Because the centroid lies on the fitted plane, substituting it into the plane equation gives:

which yields . Thus, the plane equation becomes:

The signed distance from point to the plane is defined as:

The total squared error is given by:

By minimizing under the constraint , and using the Lagrange multiplier method, a unique normal vector can be obtained as , from which the fitting plane equation is determined as:

where is the coordinate vector of any point on the plane.

In addition, to ensure that the point of maximum curvature used for subsequent arc fitting lies on the fitting plane, the fitting planemust pass through the point corresponding to the maximum curvature.

* 1. **Point Projection onto the Fitted Plane**

After obtaining the fitting plane , the centerline points of the fitted arc were orthogonally projected onto the plane to extract the geometric characteristics of the curved segment and to perform circular fitting within the plane.

For each point belonging to , a vector from the centroid of the fitted arc to that point is defined as:

This vector can be decomposed into two components:

* Normal components along the plane’s normal direction: .
* The in-plane component parallel to the plane: 。

By removing the normal component, the projected point on the fitted plane can be expressed as the three-dimensional coordinates of the point after being orthogonally projected onto the plane:

Substituting *Equation (17)* into *Equation (18)*, the explicit form becomes:

To describe the positions of these projection points on the plane, a local 2D orthogonal coordinate system is established on the fitting plane, satisfying:

In this coordinate system, the two-dimensional coordinates of the projected points are given by:

* 1. **Definition of Fitted Circles and Shape Parameters**

To determine the parameters of the fitted circle, the least-squares method was applied to the projected points obtained in the local 2D coordinate system defined on the fitted plane.

For each projected point, the circular relationship is given by:

where represents the coordinates of the circle’s center, and denotes the radius.

The sum of squared errors is then defined as:

By minimizing , the optimal parameters of the fitted circle can be obtained.

The fitted circle radiiandrepresent the curvature intensity of the bending segments and , respectively. To quantify the degree of spatial non-coplanarity between the two bending segments, the normal vectors of their corresponding fitted planes are defined asand, and the angle between them is defined as:

1. **Selection of the Optimal Fitted Arc**

As described in Section 3.1, the fitting range of the circular arc is determined by the threshold parameter . To further obtain a fitted arc that best represents the shape of the curved segment, this study calculated combinations of different values and selected the optimal arc based on the principle of minimizing fitting error.

Taking the B1 as an example, the error function between the curvature of the fitted circle and the original curvature distribution is defined as:

The error represents the deviation between the equivalent curvature of the fitted circle and the mean curvature of the original curve.

For each centerline, was varied within the range of 0.1 to 0.9, and the corresponding error function was computed to compare the distribution of fitting errors. The value that yielded the smallest error was selected as the optimal threshold parameter, denoted as . Therefore, the start and end points of the optimal fitted arc, denoted as and , these positions represent the points along the centerline where the curvature values first and last satisfy .

Accordingly, the set of points forming the optimal fitted arc can be expressed as:

Through this method, the optimal fitted arcs for both B1 and B2 segments were determined for each centerline. Subsequently, the corresponding geometric parameters were extracted using the procedures described in the previous sections, providing the basis for quantitative analysis in the following study.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Section** | **Symbol** | **Type** | **Description** |
| 1 | 1 |  | Set | Set of points representing the ICA centerline |
| 2 |  | Vector | Coordinate vector of the i-th point |
| 3 |  | Scalar | Curvature value at point equivalently expressed as along the arc-length parameterization. |
| 4 |  | Scalar | Cumulative arc length corresponding to point |
| 5 | 2 | , | Scalar | Arc-length positions of the curvature peaks B1 and B2 along the ICA centerline. |
| 6 | , | Scalar | Curvature values corresponding to the B1 and B2 curvature peaks. |
| 7 |  | Scalar | Arc length corresponding to the curvature valley proximal to the B1 peak (start of the siphon). |
| 8 |  | Scalar | Arc length corresponding to the curvature valley distal to the B2 peak (end of the siphon). |
| 9 |  | Scalar | Arc length corresponding to the curvature valley between B1 and B2 (boundary separating the two bends). |
| 10 |  | Set | Set of points within the ICA siphon, defined for |
| 11 |  | Set | Set of points corresponding to the B1 bend, defined for |
| 12 |  | Set | Set of points corresponding to the B2 bend, defined for |
| 13 | 3.1 |  | Scalar | Threshold parameter controlling the curvature-based fitting range. |
| 14 |  | Scalar | Cumulative arc length at the first point where the curvature satisfies . |
| 15 |  | Scalar | Cumulative arc length at the last point where the curvature satisfies . |
| 16 |  | Set | Set of points between and that meet the curvature condition . |
| 17 | 3.2 |  | Vector | Coordinate vector of the j-th point belonging to the set . |
| 18 |  | Vector | Centroid of all points in the set . |
| 19 |  | Scalar | Signed distance from point to the fitting plane |
| 20 |  | Scalar | Sum of squared distances (error function) used in plane fitting. , and are the components of the normal vector |
| 21 |  | Vector | Normalized normal vector of the fitted plane obtained by minimizing . |
| 22 |  | Vector | Coordinate vector of an arbitrary point on the plane |
| 23 |  | Plane | Fitted plane |
| 24 | 3.3 |  | Vector | Vector from the centroid to point . |
| 25 |  | Vector | Normal component of along the plane normal . |
| 26 |  | Vector | In-plane component of parallel to the fitted plane. |
| 27 |  | Vector | Three-dimensional coordinates of the point after being orthogonally projected onto the fitted plane |
| 28 |  | Vector | Local orthogonal basis vectors on the fitted plane |
| 29 |  | Scalar | 2D coordinates of the projected point in the plane coordinate system. |
| 30 | 3.4 |  | Scalar | Sum of squared errors between projected points and the fitted circle, used to determine optimal circle parameters. Here, and are the coordinates of the circle center, and is the circle radius. |
| 31 |  | Scalar | Radius of the fitted circle corresponding to the B1 bend. |
| 32 |  | Scalar | Radius of the fitted circle corresponding to the B2 bend. |
| 33 |  | Scalar | Angle between the fitted planes of B1 and B2, representing the degree of spatial non-coplanarity between the two bends. |
| 34 | 4 |  | Scalar | Represents the deviation between the equivalent curvature of the fitted circle and the curvature of the original curve |
| 35 |  | Scalar | The optimal threshold parameter obtained by varying α within the range of 0.1–0.9 and selecting the value that minimizes the fitting error. |
| 36 |  | Scalar | The cumulative arc-length positions of the start and end points of the optimal fitted arc, corresponding to the points along the centerline where the curvature first and last satisfies . |
| 37 |  | set | Set of points forming the optimal fitted arc. |